Univerza v Ljubljani
Fakulteta za matematiko in fiziko
Oddelek za fiziko
Smer: naravoslovna fizika

GRUPI SU(3) in SU(4) ter MEZONSKI MULTIPLETI

Seminar pri predmetu Kvantna Mehanika 2

Autor: Barbara Horvat
Mentor: prof.dr.Norma Mankoč-Borštnik

1 GRUPA SU(3) in MEZONSKI MULTIPLETI

1.1 Algebra grupe SU(3)

Grupa SU(3) ima osem generatorjev ($n^2 - 1 = 8$, kjer $n = 3$), ki smo jih zapisali kot:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Vidimo, da so vse matrike λ^i, kjer $i = 1:8$, brezsledne ter hermitske, da je pet realnih ter tri imaginarne, ter da sta dve diagonalni (λ^3 in λ^8).

Na tem mestu definiramo $\tau^{3k} = \frac{\lambda^k}{2}$ ter zapišemo algebro:

$\{\lambda^i, \lambda^j\}_- = 2if^{ijk}\lambda^k$ oz. $\{\tau^{3i}, \tau^{3j}\}_- = if^{ijk}\tau^{3k}$
$\{\lambda^i, \lambda^j\}_+ = \frac{4}{3}\delta^{ij} + 2d^{ijk}\lambda^k$

$f^{ijk} = \frac{1}{4i}Tr(\{\lambda^i, \lambda^j\}_-\lambda^k)$ ...antisimetrični tenzor
$d^{ijk} = \frac{1}{4}Tr(\{\lambda^i, \lambda^j\}_+\lambda^k)$ ...simetrični tenzor

$Tr(\lambda^i) = 0$
$Tr(\lambda^i\lambda^j) = 2\delta^{ij}$
$Tr(\{\lambda^i, \lambda^j\}_+) = 4\delta^{ij}$

Casimirja ($r = rang(SU(n = 3)) = n - 1 = 3 - 1 = 2$ predstavlja število komutirajočih operatorjev/maksimalno število hkrati diagonalnih matrik λ^i, število Casimirjev/invariant):
$C^1 = \sum_i (\tau^{3i})^2 = -\frac{2}{3}\sum_i f^{ijk}\tau^{3i}\tau^{3j}\tau^{3k}$
$C^2 = \sum_i d^{ijk}\tau^{3i}\tau 3j\tau^{3k} = C^1(2C^1 - \frac{11}{6}I)$

ijk	123	147	156	246	257	345	367	458	678
f^{ijk}	1	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$

ijk	118	146	157	228	247	256	338	344	355
d^{ijk}	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$

ijk	366	377	448	558	668	778	888
d^{ijk}	-$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{2\sqrt{3}}$	-$\frac{1}{2\sqrt{3}}$	-$\frac{1}{2\sqrt{3}}$	-$\frac{1}{2\sqrt{3}}$	-$\frac{1}{2\sqrt{3}}$

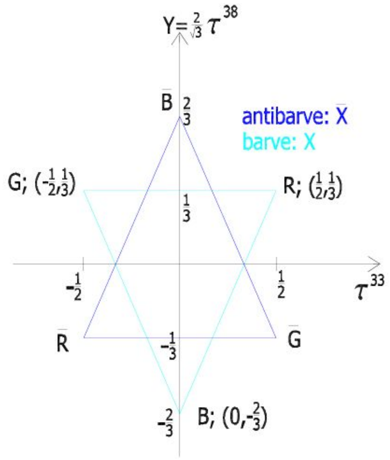
Pimer izračuna komponent simetričnega ter antisimetričnega tenzorja je v prilogi.

1.2 (τ^{33}, $\frac{2}{\sqrt{3}}\tau^{38}$) ravnina

Hilbertov prostor nad katerimi delujejo matrike λ^k razpenjajo vektorji:

$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, kjer R predstavlja rdečo barvo, G zeleno, B pa modro barvo. Z drugimi besedami: grupo SU(3) uporabimo za opis barvnega naboja. Sedaj pa delujemo na te bazne vektorje z diagonalnima matrikama τ^{33} in τ^{38} ter na absciso τ^{33} nanašamo lastne vrednosti dobljene z delovanjem matrike τ^{33} na baznih vektorjih, na ordinato $Y = \frac{2}{\sqrt{3}}\tau^{38}$ pa lastne vrednosti dobljene z delovanjem matrike $\frac{2}{\sqrt{3}}\tau^{38}$ na baznih vektorjih. Upoštevamo tudi, da za antibarve le spremenimo predznak lastnim vrednostim. Tako lahko zapišemo za barve:

$$
\begin{aligned}
\tau^{33}\begin{pmatrix}1\\0\\0\end{pmatrix} &= \tfrac{1}{2}\begin{pmatrix}1\\0\\0\end{pmatrix},\ \tau^{38}\begin{pmatrix}1\\0\\0\end{pmatrix} = \tfrac{1}{2\sqrt{3}}\begin{pmatrix}1\\0\\0\end{pmatrix}\\
\tau^{33}\begin{pmatrix}0\\1\\0\end{pmatrix} &= \text{-}\tfrac{1}{2}\begin{pmatrix}0\\1\\0\end{pmatrix},\ \tau^{38}\begin{pmatrix}0\\1\\0\end{pmatrix} = \tfrac{1}{2\sqrt{3}}\begin{pmatrix}0\\1\\0\end{pmatrix}\\
\tau^{33}\begin{pmatrix}0\\0\\1\end{pmatrix} &= 0\begin{pmatrix}0\\0\\1\end{pmatrix},\ \tau^{38}\begin{pmatrix}0\\0\\1\end{pmatrix} = \text{-}\tfrac{1}{\sqrt{3}}\begin{pmatrix}0\\0\\1\end{pmatrix}.
\end{aligned}
$$



Slika 1: Barve ter antibarve v ravnini (τ^{33},Y).

Seveda dobimo lastne vrednosti za antibarve z obrnjenim predznakom glede na lastne vrednosti za barve tako, da uporabimo na baznih vektorjih za antibarve (so isti kot za barve, le da razpenjajo popolnoma drug Hilbertov prostor) matrike λ za antibarve. Le-te pa dobimo s pomočjo operatorja konjugacije naboja: $\mathcal{C} = i\gamma^2 Kc$, kjer K naredi konjugacijo, c pa spremeni predznak. Torej:

$$
\begin{pmatrix}\tau_c^{31}\\ \tau_c^{32}\\ \tau_c^{33}\\ \tau_c^{34}\\ \tau_c^{35}\\ \tau_c^{36}\\ \tau_c^{37}\\ \tau_c^{38}\end{pmatrix} = \mathcal{C}\begin{pmatrix}\tau_r^{31}\\ \tau_i^{32}\\ \tau_r^{33}\\ \tau_r^{34}\\ \tau_i^{35}\\ \tau_r^{36}\\ \tau_i^{37}\\ \tau_r^{38}\end{pmatrix}\mathcal{C}^{-1} = \begin{pmatrix}\text{-}\tau_r^{31}\\ \tau_i^{32}\\ \text{-}\tau_r^{33}\\ \text{-}\tau_r^{34}\\ \tau_i^{35}\\ \text{-}\tau_c^{36}\\ \tau_i^{37}\\ \text{-}\tau_c^{38}\end{pmatrix},
$$

kjer indeks r/i označuje matriko z realnimi/imaginarnimi koeficienti, indeks c pa matriko za "antidelce". Le-te pa morajo ponovno zadoščati komutacijskim pravilom. Preverimo za en primer:

- $\{\tau^{35}, \tau^{36}\}_- = if^{561}\tau^{31} = -\frac{1}{2}i\tau^{31}$ za "delce"
- $\{\tau_c^{35}, \tau_c^{36}\}_- = if^{561}\tau_c^{31} = -\frac{1}{2}i\tau_c^{31}$ za "antidelce"
  Leva stran je enaka: $\{\tau^{35}, -\tau^{36}\}_- = -\{\tau^{35}, \tau^{36}\}_- = -(-\frac{1}{2}i\tau^{31}) = \frac{1}{2}i\tau^{31}$,
  desna stran pa:$-\frac{1}{2}i\tau_c^{31} = -\frac{1}{2}i(-\tau^{31}) = \frac{1}{2}i\tau^{31}$.

  Torej je leva stran enaka desni oz. komutacijska pravila veljajo tudi za primer "antidelcev".

1.3 Generatorji podgrup grupe SU(3)

Vpeljemo novo bazo $|\tau^{33}Y>$ za katero velja:
$\hat{\tau}^{33}|\tau^{33}Y>=\tau^{33}|\tau^{33}Y>$
$\hat{Y}|\tau^{33}Y>=Y|\tau^{33}Y>$.

Poiskati želimo generatorje podgrup grupe SU(3) (zaprejo algebro), saj nas bo zanimalo, kako le ti delujejo na bazi $|\tau^{33}Y>$, da bomo lahko zgradili multiplete.

Vemo, da za $i,j,k=1,2,3$ velja $\{\tau^{3i},\tau^{3j}\}_-=i\epsilon^{ijk}\tau^{3k}$. To je enako kot pri grupi SU(2). Tako lahko zapišemo $\hat{\tau}^{\pm}=\hat{\tau}^{31}\pm i\hat{\tau}^{32}$, kar je eden izmed generatorjev. Na podoben način lahko zapišemo še ostale generatorje (preverili bomo seveda tudi, če zaprejo algebro).

Generatorji so torej:
$\hat{\tau}^{\pm}=\hat{\tau}^{31}\pm i\hat{\tau}^{32}$
$\hat{\tau}^{33}$
$\hat{V}^{\pm}=\hat{\tau}^{34}\pm i\hat{\tau}^{35}$
$\hat{U}^{\pm}=\hat{\tau}^{36}\pm i\hat{\tau}^{37}$
$\hat{Y}=\frac{2}{\sqrt{3}}\hat{\tau}^{38}$.

Njihovi komutatorji:
$\{\hat{\tau}^{33},\hat{\tau}^{\pm}\}_-=\pm\hat{\tau}^{\pm}$ $\quad\{\hat{Y},\hat{\tau}^{\pm}\}_-=0$
$\{\hat{\tau}^{33},\hat{V}^{\pm}\}_-=\pm\frac{1}{2}\hat{V}^{\pm}$ $\quad\{\hat{Y},\hat{V}^{\pm}\}_-=\pm\hat{V}^{\pm}$
$\{\hat{\tau}^{33},\hat{U}^{\pm}\}_-=\mp\frac{1}{2}\hat{U}^{\pm}$ $\quad\{\hat{Y},\hat{U}^{\pm}\}_-=\pm\hat{U}^{\pm}$

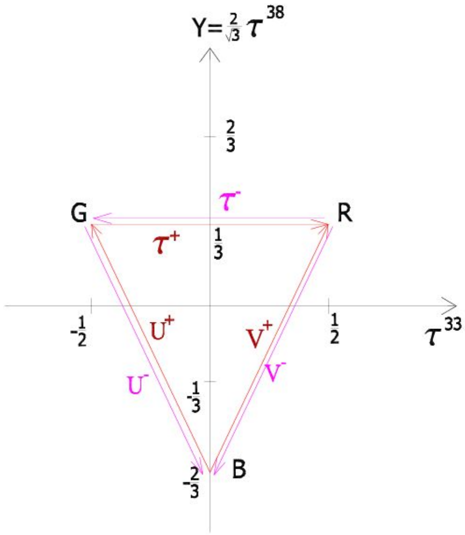
$\{\hat{\tau}^{+},\hat{\tau}^{-}\}_-=2\hat{\tau}^{33}$
$\{\hat{V}^{+},\hat{V}^{-}\}_-=\frac{3}{2}\hat{Y}+\hat{\tau}^{33}:=2\hat{V}^{33}$ $\quad\{\hat{V}^{\pm},\hat{V}^{33}\}_-=\mp\hat{V}^{\pm}$
$\{\hat{U}^{+},\hat{U}^{-}\}_-=\frac{3}{2}\hat{Y}-\hat{\tau}^{33}:=2\hat{U}^{33}$ $\quad\{\hat{U}^{\pm},\hat{U}^{33}\}_-=\mp\hat{U}^{\pm}$
iz katerih je očitno, da zaprejo algebro.
Primer izračuna komutatorjev je v prilogi.

Podgrupe grupe SU(3) so grupe SU(2), ki pa v našem primeru niso invariantne. Za njih pa lahko zapišemo:
$\{\hat{\tau}^{i},\hat{\tau}^{j}\}_-=i\epsilon^{ijk}\hat{\tau}^{k}$, kjer $\hat{\tau}^{i}=\hat{\tau}^{3i}$ za $i,j,k=1,2,3$
$\{\hat{V}^{i},\hat{V}^{j}\}_-=i\epsilon^{ijk}\hat{V}^{k}$, kjer $\hat{V}^{i,j}=\hat{\tau}^{3i,3j}$ za $i,j=4,5$ oz. $\hat{V}^{k}=\hat{V}^{33}$
$\{\hat{U}^{i},\hat{U}^{j}\}_-=i\epsilon^{ijk}\hat{U}^{k}$, kjer $\hat{U}^{i,j}=\hat{\tau}^{3i,3j}$ za $i,j=6,7$ oz. $\hat{U}^{k}=\hat{U}^{33}$.

Kako operatorji delujejo na bazi $|\tau^{33}\ Y>$ (da ugotovimo, kako delujejo operatorji $\hat{\tau}^{\pm}$, $\hat{V}^{\pm}$, $\hat{U}^{\pm}$ na bazi, moramo na npr. $\hat{\tau}^{\pm}|\tau^{33}\ Y>$ delovati enkrat z operatorjem $\hat{\tau}^{33}$ drugič pa z $\hat{Y}$; Primer izračuna spodnjih izrazov je v prilogi.):

- $\hat{\tau}^{33}|\tau^{33}\ Y>=\tau^{33}|\tau^{33}\ Y>$
  $\hat{Y}|\tau^{33}\ Y>=Y|\tau^{33}\ Y>$
- $\hat{\tau}^{33}\hat{\tau}^{\pm}|\tau^{33}\ Y>=(\tau^{33}\pm 1)\hat{\tau}^{\pm}|\tau^{33}\ Y>$
  $\hat{Y}\hat{\tau}^{\pm}|\tau^{33}\ Y>=Y\hat{\tau}^{\pm}|\tau^{33}\ Y>$
- $\hat{\tau}^{33}\hat{V}^{\pm}|\tau^{33}\ Y>=(\tau^{33}\pm\frac{1}{2})\hat{V}^{\pm}|\tau^{33}\ Y>$
  $\hat{Y}\hat{V}^{\pm}|\tau^{33}\ Y>=(Y\pm 1)\hat{V}^{\pm}|\tau^{33}\ Y>$
- $\hat{\tau}^{33}\hat{U}^{\pm}|\tau^{33}\ Y>=(\tau^{33}\mp\frac{1}{2})\hat{U}^{\pm}|\tau^{33}\ Y>$
  $\hat{Y}\hat{U}^{\pm}|\tau^{33}\ Y>=(Y\pm 1)\hat{U}^{\pm}|\tau^{33}\ Y>$

Oz. na sliki 1 na strani 3 bi izgledalo to:



Slika 2: Preslikave barv z operatorji $\tau^{\pm}$, $V^{\pm}$, $U^{\pm}$.

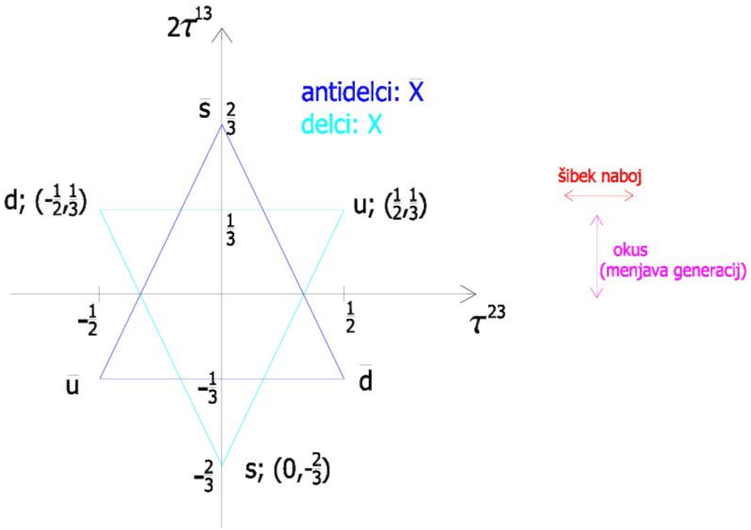
saj:

- $\hat{\tau}^-(\frac{1}{2}, \frac{1}{3}) = (-\frac{1}{2}, \frac{1}{3})$
  $\hat{\tau}^+(-\frac{1}{2}, \frac{1}{3}) = (\frac{1}{2}, \frac{1}{3})$
- $\hat{V}^-(\frac{1}{2}, \frac{1}{3}) = (0, -\frac{2}{3})$
  $\hat{V}^+(0, -\frac{2}{3}) = (\frac{1}{2}, \frac{1}{3})$
- $\hat{U}^-(-\frac{1}{2}, \frac{1}{3}) = (0, -\frac{2}{3})$
  $\hat{U}^+(0, -\frac{2}{3}) = (-\frac{1}{2}, \frac{1}{3})$

Za ostala stanja dajo ti operatorji nič.

1.4 Barve "zamenjamo" z delci

Za razlago delcev rečemo, da lastna vrednost operatorja $\hat{Y} = \frac{2}{\sqrt{3}}\hat{\tau}^{38}$ za barve predstavlja hipernaboj ($2\hat{\tau}^{13}$) za delce, lastna vrednost operatorja $\hat{\tau}^{33}$ za barve pa tretjo komponento izospina ($\hat{\tau}^{23}$) za delce. Tako povežemo grupo SU(3) z okusom ter šibkim nabojem; d u s predstavljajo triplet. Lepše se vidi na sliki 3:



Slika 3: Delci ter antidelci v ravnini $(\tau^{33}, Y = \frac{2}{\sqrt{3}}\tau^{38})$.

1.5 Gradnja mezonov

Velja:

$$\hat{\vec{\tau}} = \sum_{delci} \hat{\vec{\tau}}^{delci} + \sum_{antidelci} \hat{\vec{\tau}}^{antidelci}.$$

S pomočjo te enačbe lahko sedaj zgradimo mezone, ki so sestavljeni iz kvarka ter antikvarka oz. delca ter antidelca.

Spomnimo se še, kakšna sta hipernaboj ter tretja komponenta izospina za kvarke u d s:

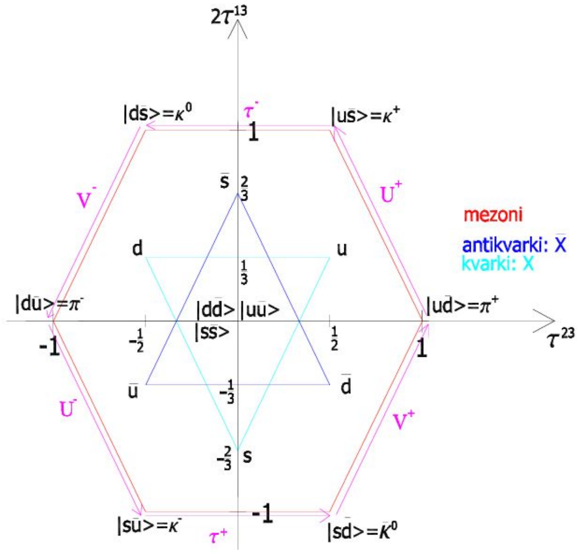
kvarki	$Y = 2\tau^{13}$	τ^{23}	Q
u	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
d	$\frac{1}{3}$	-$\frac{1}{2}$	-$\frac{1}{3}$
s	-$\frac{2}{3}$	0	-$\frac{1}{3}$

kjer je $Q = \tau^{13} + \tau^{23}$.

Tako za mezone dobimo:

mezoni $(\tau^{33}, \frac{2}{\sqrt{3}}\tau^{38}) \rightarrow (\tau^{23}, 2\tau^{13})$	$Y = 2\tau^{13}$	τ^{23}	Q
$\kappa^+ = \|u > \|\bar{s} >= (\frac{1}{2}, \frac{1}{3})(0, \frac{2}{3})$	1	$\frac{1}{2}$	1
$\kappa^0 = \|d > \|\bar{s} >= (-\frac{1}{2}, \frac{1}{3})(0, \frac{2}{3})$	1	-$\frac{1}{2}$	0
$\|s > \|\bar{s} >= (0, -\frac{2}{3})(0, \frac{2}{3})$	0	0	0
$\pi^+ = \|u > \|\bar{d} >= (\frac{1}{2}, \frac{1}{3})(\frac{1}{2}, -\frac{1}{3})$	0	1	1
$\|d > \|\bar{d} >= (-\frac{1}{2}, \frac{1}{3})(\frac{1}{2}, -\frac{1}{3})$	0	0	0
$\bar{\kappa}^0 = \|s > \|\bar{d} >= (0, -\frac{2}{3})(\frac{1}{2}, -\frac{1}{3})$	-1	$\frac{1}{2}$	0
$\|u > \|\bar{u} >= (\frac{1}{2}, \frac{1}{3})(-\frac{1}{2}, -\frac{1}{3})$	0	0	0
$\pi^- = \|d > \|\bar{u} >= (-\frac{1}{2}, \frac{1}{3})(-\frac{1}{2}, -\frac{1}{3})$	0	-1	-1
$\kappa^- = \|s > \|\bar{u} >= (0, -\frac{2}{3})(-\frac{1}{2}, -\frac{1}{3})$	-1	-$\frac{1}{2}$	-1

kar lahko narišemo v $(\tau^{23}, 2\tau^{13})$ ravnini:



Slika 4: Mezoni v ravnini $(\tau^{23}, 2\tau^{13})$.

Tako za delce kot za barve velja (iz slik 2 ter 4):
$\hat{\tau}^{+}|d>=|u>$
$\hat{V}^{+}|s>=|u>$
$\hat{U}^{+}|s>=|d>$

Za antidelce pa velja:
$\hat{\tau}^{+}|\bar{u}>=-|\bar{d}>$
$\hat{V}^{+}|\bar{u}>=-|\bar{s}>$
$\hat{U}^{+}|\bar{d}>=-|\bar{s}>$

Minus pri izrazih za antidelce dobimo, ker delujemo na stanja antidelcev z operatorjem za delce; operator za delce ter antidelce pa povežemo z:
$\vec{\tau}_c = \mathcal{C}\vec{\tau}\mathcal{C}^{-1}$
$\hat{\tau}_c^{\pm} = \mathcal{C}\hat{\tau}^{\pm}\mathcal{C}^{-1} = \mathcal{C}(\hat{\tau}^{31} \pm i\hat{\tau}^{32})\mathcal{C}^{-1} = \hat{\tau}_c^{31} \pm (-i)\hat{\tau}_c^{32} = -\hat{\tau}^{31} \mp i\hat{\tau}^{32} = -\hat{\tau}^{\pm}$
$\hat{\tau}_c^{33} = -\hat{\tau}^{33}$
$\hat{V}_c^{\pm} = \hat{\tau}_c^{34} \mp i\hat{\tau}_c^{35} = -\hat{\tau}^{34} \mp i\hat{\tau}^{35} = -\hat{V}^{\pm}$
$\hat{U}_c^{\pm} = \hat{\tau}_c^{36} \mp i\hat{\tau}_c^{37} = -\hat{\tau}^{36} \mp i\hat{\tau}^{37} = -\hat{U}^{\pm}$
$\hat{Y}_c = \frac{2}{\sqrt{3}}\hat{\tau}_c^{38} = -\frac{2}{\sqrt{3}}\hat{\tau}^{38} = -\hat{Y}$.

Primer komutatorja med zgornjimi operatorji za antidelce je v prilogi (ven padejo zgornje zveze). Temu je dodano še delovanje enega izmed operatorjev za antidelce na bazi. S pomočjo tega pa določimo preslikave z operatorji za delce med antidelci.

Poiščimo še ostala stanja (imamo že šest mezonov, manjkajo pa še trije, saj kombiniramo v pare tri različne kvarke):

1. Mezon π^0 (del tripleta glede na $\hat{\tau}^{\pm}$; glej poglavje 6)

   $\hat{\tau}^{+}|d\bar{u}>=(\hat{\tau}^{+}|d>)|\bar{u}>+|d>\hat{\tau}^{+}|\bar{u}>=|u\bar{u}>-|d\bar{d}>\propto \underline{\frac{1}{\sqrt{2}}(|u\bar{u}>-|d\bar{d}>)=\pi^0}$

2. Mezon $\eta^{`} = \alpha|u\bar{u}>+\beta|d\bar{d}>+\gamma|s\bar{s}>$ (singlet):

   Upoštevajmo ortogonalnost: $<\pi^0|\eta^{`}>=0$
   $<\alpha u\bar{u}+\beta d\bar{d}+\gamma s\bar{s}|\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})>=\frac{1}{\sqrt{2}}(\alpha-\beta)=0$
   Torej: $\alpha=\beta$.

   Da pa je $\eta^{`}$ res singlet, pa moramo zahtevati še: $\hat{\tau}^{\pm}|\eta^{`}>=\hat{V}^{\pm}|\eta^{`}>=\hat{U}^{\pm}|\eta^{`}>=0$
   $\hat{V}^{+}|\eta^{`}>=\hat{V}^{+}|\alpha u\bar{u}+\alpha d\bar{d}+\gamma s\bar{s}>=-\alpha|u\bar{s}>+0+\gamma|u\bar{s}>=0$
   Torej: $\alpha=\gamma$.
   $\hat{\tau}^{+}|\eta^{`}>=\hat{\tau}^{+}|u\bar{u}+d\bar{d}+s\bar{s}>=-|u\bar{d}>+|u\bar{d}>+0=0$

$\hat{U}^{+}|\eta^{`}>=\hat{U}^{+}|u\bar{u}+d\bar{d}+s\bar{s}>=0-|d\bar{s}>+|d\bar{s}>=0$

Seveda tudi če delujemo z operatorji $\hat{\tau}^{-}$, $\hat{V}^{-}$, $\hat{U}^{-}$ na stanju $|\eta^{`}>$, dobimo 0.

$\eta^{`}=\frac{1}{\sqrt{3}}(|u\bar{u}>+|d\bar{d}>+|s\bar{s}>)$

3. Mezon $\eta^0=a|u\bar{u}>+b|d\bar{d}>+c|s\bar{s}>$ (del dveh tripletov: glede na $\hat{U}^{\pm}$, $\hat{V}^{\pm}$; glej poglavje 6):

   Upoštevajmo ortogonalnost: $<\eta^0|\pi^0>=0$

   $<au\bar{u}+bd\bar{d}+cs\bar{s}|\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})>=\frac{1}{\sqrt{2}}(a-b)=0$

   Torej: $a=b$.

   Upoštevajmo ortogonalnost: $<\eta^0|\eta^{`}>=0$

   $<au\bar{u}+ad\bar{d}+cs\bar{s}|\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})>=\frac{1}{\sqrt{2}}(a+a+c)=0$

   Torej: $c=-2a$.

   $\eta^0=\frac{1}{\sqrt{6}}(|u\bar{u}>+|d\bar{d}>-2|s\bar{s}>)$

1.6 Razvrstitev mezonov v singlet...

Sedaj pa nas zanimajo mezonski dubleti, tripleti, oktet ter singlet, ki jih ugotavljamo s pomočjo operatorjev generatorjev podgrup grupe SU(3).

- Operator $\hat{\tau}^{\pm}$ (dovolj, da pogledamo le $\hat{\tau}^{+}$):

  1. Dubleta

     Dublet $|d\bar{s}>$, $|u\bar{s}>$ (κ^0, κ^+):
     $\hat{\tau}^{+}|d\bar{s}>=(\hat{\tau}^{+}|d>)|\bar{s}>+|d>\hat{\tau}^{+}|\bar{s}>=|u>|\bar{s}>+0=|u\bar{s}>$
     $\hat{\tau}^{+}|u\bar{s}>=0$

     Dublet $|s\bar{u}>$, $|s\bar{d}>$ (κ^-, $\bar{\kappa}^0$):
     $\hat{\tau}^{+}|s\bar{u}>=-|s\bar{d}>\propto|s\bar{d}>$
     $\hat{\tau}^{+}|s\bar{d}>=0$

  2. Triplet $|d\bar{u}>$, $\frac{1}{\sqrt{2}}(|u\bar{u}>-|d\bar{d}>)$, $|u\bar{d}>$ (π^-, π^0, π^+):

     $\hat{\tau}^{+}|d\bar{u}>=|u\bar{u}>-|d\bar{d}>\propto\frac{1}{\sqrt{2}}(|u\bar{u}>-|d\bar{d}>)$
     $\hat{\tau}^{+}(|u\bar{u}>-|d\bar{d}>)=-2|u\bar{d}>\propto|u\bar{d}>$
     $\hat{\tau}^{+}|u\bar{d}>=0$

- Operator $\hat{V}^{\pm}$:

  1. Dubleta

     Dublet $|d\bar{u}>$, $|d\bar{s}>$ (π^-, κ^0):
     $\hat{V}^{+}|d\bar{u}>=-|d\bar{s}>\propto|d\bar{s}>$
     $\hat{V}^{+}|d\bar{s}>=0$

     Dublet $|s\bar{d}>$, $|u\bar{d}>$ ($\bar{\kappa}^0$, π^+):
     $\hat{V}^{+}|s\bar{d}>=|u\bar{d}>$
     $\hat{V}^{+}|u\bar{d}>=0$

2. <u>Tripleta</u>

   Triplet $|s\bar{u}>$, $\frac{1}{\sqrt{2}}(|u\bar{u}> - |d\bar{d}>)$, $|u\bar{s}>$ $(\kappa^-,\ \pi^0,\ \kappa^+)$:
   $\hat{V}^+(|u\bar{u}> - |d\bar{d}>) = -|u\bar{s}> \propto |u\bar{s}>$
   $\hat{V}^+|u\bar{s}>= 0$
   $\hat{V}^-(|u\bar{u}> - |d\bar{d}>) = |s\bar{u}>$
   $\hat{V}^-|s\bar{u}>= 0$

   Triplet $|s\bar{u}>$, $\frac{1}{\sqrt{6}}(|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>)$, $|u\bar{s}>$ $(\kappa^-,\ \eta^0,\ \kappa^+)$:
   $\hat{V}^+(|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>) = -3|u\bar{s}> \propto |u\bar{s}>$
   $\hat{V}^+|u\bar{s}>= 0$
   $\hat{V}^-(|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>) = 3|s\bar{u}> \propto |s\bar{u}>$
   $\hat{V}^-|s\bar{u}>= 0$

- Operator $\hat{U}^{\pm}$:

1. <u>Dubleta</u>

   Dublet $|u\bar{d}>$, $|u\bar{s}>$ $(\pi^+,\ \kappa^+)$:
   $\hat{U}^+|u\bar{d}>= -|u\bar{s}> \propto |u\bar{s}>$
   $\hat{U}^+|u\bar{s}>= 0$

   Dublet $|s\bar{u}>$, $|d\bar{u}>$ $(\kappa^-,\ \pi^-)$:
   $\hat{U}^+|s\bar{u}>= |d\bar{u}>$
   $\hat{U}^+|d\bar{u}>= 0$

2. <u>Triplet</u>

   Triplet $|s\bar{d}>$, $\frac{1}{\sqrt{2}}(|u\bar{u}> - |d\bar{d}>)$, $|u\bar{s}>$ $(\bar{\kappa}^0,\ \pi^0,\ \kappa^0)$:
   $\hat{U}^+(|u\bar{u}> - |d\bar{d}>) = |d\bar{s}>$
   $\hat{U}^+|d\bar{s}>= 0$
   $\hat{U}^-(|u\bar{u}> - |d\bar{d}>) = -|s\bar{d}> \propto |s\bar{d}>$
   $\hat{U}^-|s\bar{d}>= 0$

   Triplet $|s\bar{d}>$, $\frac{1}{\sqrt{6}}(|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>)$, $|d\bar{s}>$ $(\bar{\kappa}^0,\ \eta^0,\ \kappa^0)$:
   $\hat{U}^+(|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>) - 3|d\bar{s}> \propto |d\bar{s}>$
   $\hat{U}^+|d\bar{s}>= 0$
   $\hat{U}^-(|u\bar{u}> + |d\bar{d}> - 2|s\bar{s}>) = 3|s\bar{d}> \propto |s\bar{d}>$
   $\hat{U}^-|s\bar{d}>= 0$

Vse delce, ki gradijo dublete oz. triplete lahko združimo v oktet glede na operatorje $\hat{\tau}^{\pm}$, $\hat{V}^{\pm}$, $\hat{U}^{\pm}$ (s pomočjo le teh lahko iz enega stanja iz okteta zgeneriramo še ostalih sedem). Oktet je potemtakem sestavljen iz: π^-, π^0, π^+, κ^0, κ^+, κ^-, $\bar{\kappa}^0$ ter η^0. Z operatorji $\hat{\tau}^{\pm}$, $\hat{V}^{\pm}$ ter $\hat{U}^{\pm}$ pa ne moremo zgenerirati stanja $\eta^{`}$ iz stanj, ki so del okteta, torej predstavlja $\eta^{`}$ singlet.

Da imamo med SU(3) multipleti, ki jih sestavimo iz SU(2) multipletov oz. T-multipletov (so vzoredni osi τ^{33}), en singlet in en oktet vidimo tudi iz tega:

$$[3]\otimes[\bar{3}] = ([2]^{\frac{1}{3}}\oplus[1]^{-\frac{2}{3}})\otimes([2]^{-\frac{1}{3}}\oplus[1]^{\frac{2}{3}}) = ([2]\otimes[2])^0\oplus[2]^{-1}\oplus[2]^1\oplus[1]^0 = [3]^0\oplus[1]^0\oplus[2]^{-1}\oplus[2]^1\oplus[1]^0 =$$
$$= [8]\oplus[1]$$

Kjer smo upoštevali, da je $[3] = [2]^{\frac{1}{3}} \oplus [1]^{-\frac{2}{3}}$, $[2]\otimes[2] = [3]\oplus[1]$ ter da indeks v eksponentu predstavlja lastno vrednost operatorja $\hat{Y}$.

2 GRUPA SU(4) in MEZONSKI MULTIPLETI

2.1 Algebra grupe SU(4)

Grupa SU(4) ima 15 generatorjev :

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^9 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{10} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \lambda^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\lambda^{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \lambda^{14} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \quad \lambda^{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}.$$

Vidimo , da so vse matrike brezsledne ter da so tri diagonalne, kar ustreza številu Casimirjevih operatorjev oz. maksimalnemu številu hkrati diagonalnih matrik λ^i.

Velja $\tau^{4k} = \frac{\lambda^k}{2}$ ter:

$\{\lambda^i, \lambda^j\}_- = 2if^{ijk}\lambda^k$ oz. $\{\tau^{4i}, \tau^{4j}\}_- = if^{ijk}\tau^{4k}$
$\{\lambda^i, \lambda^j\}_+ = \delta^{ij} + 2d^{ijk}\lambda^k$

$f^{ijk} = \frac{1}{4i}Tr(\{\lambda^i, \lambda^j\}_-\lambda^k)$ ...antisimetrični tenzor
$d^{ijk} = \frac{1}{4}Tr(\{\lambda^i, \lambda^j\}_+\lambda^k)$ ...simetrični tenzor

$Tr(\lambda^i) = 0$
$Tr(\lambda^i\lambda^j) = 2\delta^{ij}$
$Tr(\{\lambda^i, \lambda^j\}_+) = 4\delta^{ij}$

ijk	123	147	156	19,12	1,10,11	246	257	29,11	2,10,12	345	367	39,10	3,11,12
f^{ijk}	1	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$

ijk	458	49,14	4,10,13	59,13	5,10,14	678	6,11,14	6,12,13	7,11,13	7,12,14	89,10
f^{ijk}	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$

ijk	8,11,12	8,13,14	9,10,15	11,12,15	13,14,15
f^{ijk}	$\frac{1}{2\sqrt{3}}$	-$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}$

ijk	118	1,1,15	146	157	19,11	1,10,12	228	22,15	247	256	29,12	2,10,11	338
d^{ijk}	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$

ijk	33,15	344	355	366	377	399	3,10,10	3,11,11	3,12,12	448	44,15	49,13
d^{ijk}	$\frac{1}{\sqrt{6}}$	$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{2}$

ijk	4,10,14	558	55,15	59,14	5,10,13	668	66,15	6,11,13	6,12,14	778	77,15
d^{ijk}	$\frac{1}{2}$	-$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{6}}$

ijk	7,11,14	7,12,13	888	88,15	899	8,10,10	8,11,11	8,12,12	8,13,13	8,14,14	99,15
d^{ijk}	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	-$\frac{1}{\sqrt{3}}$	-$\frac{1}{\sqrt{3}}$	-$\frac{1}{\sqrt{6}}$

ijk	10,10,15	11,11,15	12,12,15	13,13,15	14,14,15	15,15,15
d^{ijk}	-$\frac{1}{\sqrt{6}}$	-$\frac{1}{\sqrt{6}}$	-$\frac{1}{\sqrt{6}}$	-$\frac{1}{\sqrt{6}}$	-$\frac{1}{\sqrt{6}}$	-$\frac{1}{\sqrt{2/3}}$

2.2 (τ^{43}, Y_4, C_4) prostor

Hilbertov prostor nad katerimi delujejo matrike λ^k razpenjajo vektorji:

$$|u>=\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\quad |d>=\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\quad |s>=\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\quad |c>=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}.$$

Po analogiji prvega dela seminarja (grupa SU(3)) kar rečemo, da $|u>$ predstavlja stanje kvarka u, $|d>$ predstavlja stanje kvarka d ... Na te bazne vektorje pa delujemo z operatorji $\hat{\tau}^{43}$, $\hat{Y}_4$ in $\hat{C}_4$, kjer sta:

$$\hat{Y}_4 = \frac{2}{\sqrt{3}}\hat{\tau}^{48} + \frac{1}{12}(I - 2\sqrt{6}\hat{\tau}^{415})$$

$$\hat{C}_4 = 2(\hat{\tau}^{43})^2 - \frac{1}{\sqrt{3}}\hat{\tau}^{48} - \sqrt{\frac{8}{3}}\hat{\tau}^{415}.$$

Tako lahko zapišemo:

kvarki	τ^{43}	Y_4	C_4	Q_4	S_4	B
u	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}$	0	$\frac{1}{3}$
d	-$\frac{1}{2}$	$\frac{1}{3}$	0	-$\frac{1}{3}$	0	$\frac{1}{3}$
s	0	-$\frac{2}{3}$	0	-$\frac{1}{3}$	-1	$\frac{1}{3}$
c	0	$\frac{1}{3}$	1	$\frac{2}{3}$	0	$\frac{1}{3}$

kjer je τ^{43} izospin, $Y_4(= B + S_4)$ hipernaboj, C_4 čudnost, Q_4 naboj, S_4 šarm, B pa barionsko število oz.:
$\hat{Q}_4 = \hat{\tau}^{43} + \frac{\hat{Y}_4}{2} + \frac{\hat{C}_4}{2}$
$\hat{S}_4 = -2(\hat{\tau}^{43})^2 + \sqrt{3}\hat{\tau}^{48}$.

Lastne vrednosti za antidelce imajo obrnjen predznak glede na lastne vrednosti za delce. Matrike τ_c^{4i} pa dobimo s pomočjo operatorja konjugacije naboja: $\mathcal{C} = i\gamma^2 Kc$, kjer K naredi konjugacijo, c pa spremeni predznak. Torej:

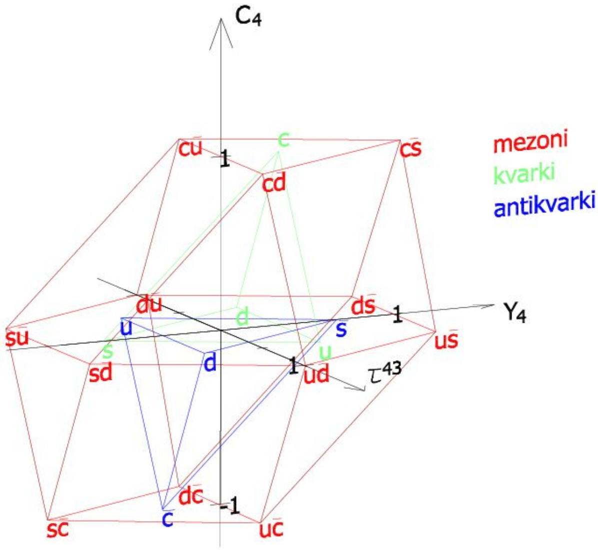
$$\begin{pmatrix}\tau_c^{41}\\\tau_c^{42}\\\tau_c^{43}\\\tau_c^{44}\\\tau_c^{45}\\\tau_c^{46}\\\tau_c^{47}\\\tau_c^{48}\\\tau_c^{49}\\\tau_c^{410}\\\tau_c^{411}\\\tau_c^{412}\\\tau_c^{413}\\\tau_c^{414}\\\tau_c^{415}\end{pmatrix} = \mathcal{C}\begin{pmatrix}\tau_r^{41}\\\tau_i^{42}\\\tau_r^{43}\\\tau_r^{44}\\\tau_i^{45}\\\tau_r^{46}\\\tau_i^{47}\\\tau_r^{48}\\\tau_r^{49}\\\tau_i^{410}\\\tau_r^{411}\\\tau_i^{412}\\\tau_r^{413}\\\tau_i^{414}\\\tau_r^{415}\end{pmatrix}\mathcal{C}^{-1} = \begin{pmatrix}-\tau_r^{41}\\\tau_i^{42}\\-\tau_r^{43}\\-\tau_r^{44}\\\tau_i^{45}\\-\tau_r^{46}\\\tau_i^{47}\\-\tau_r^{48}\\-\tau_r^{49}\\\tau_i^{410}\\-\tau_r^{411}\\\tau_i^{412}\\-\tau_r^{413}\\\tau_i^{414}\\-\tau_r^{415}\end{pmatrix},$$

kjer indeks r/i označuje matriko z realnimi/imaginarnimi koeficienti, indeks c pa matriko za antidelce.

Tako za mezone dobimo:

mezoni	τ^{43}	Y_4	S_4	C_4	Q_4
$\kappa^+ = \|u\bar{s}>$	$\frac{1}{2}$	1	1	0	1
$\kappa^0 = \|d\bar{s}>$	-$\frac{1}{2}$	1	1	0	0
$\|s\bar{s}>$	0	0	0	0	0
$F^+ = \|c\bar{s}>$	0	1	1	1	1
$\pi^+ = \|u\bar{d}>$	1	0	0	0	1
$\|d\bar{d}>$	0	0	0	0	0
$\bar{\kappa}^0 = \|s\bar{d}>$	$\frac{1}{2}$	-1	-1	0	0
$D^+ = \|c\bar{d}>$	$\frac{1}{2}$	0	0	1	1
$\|u\bar{u}>$	0	0	0	0	0
$\pi^- = \|d\bar{u}>$	-1	0	0	0	-1
$\kappa^- = \|s\bar{u}>$	-$\frac{1}{2}$	-1	-1	0	-1
$D^0 = \|c\bar{u}>$	-$\frac{1}{2}$	0	0	1	0
$\bar{D}^0 = \|u\bar{c}>$	$\frac{1}{2}$	0	0	-1	0
$D^- = \|d\bar{c}>$	-$\frac{1}{2}$	0	0	-1	-1
$F^- = \|s\bar{c}>$	0	-1	-1	-1	-1
$\|c\bar{c}>$	0	0	0	0	0

Tabelo lahko narišemo v prostoru (τ^{43}, Y_4, C_4), kjer na os x nanašamo lastne vrednosti operatorja $\hat{\tau}^{43}$, na os y lastne vrednosti operatorja $\hat{Y}_4$, na os z pa lastne vrednosti operatorja $\hat{C}_4$.



Slika 5: Mezoni v prostoru (τ^{43}, Y_4, C_4).

2.3 Generatorji podgrup grupe SU(4)

Generatorji podgrup grupe SU(4), ki zaprejo algebro, so:

$\hat{\tau}_4^{\pm} = \hat{\tau}^{41} \pm i\hat{\tau}^{42}$

$\hat{\tau}^{43}$

$\hat{V}_4^{\pm} = \hat{\tau}^{44} \pm i\hat{\tau}^{45}$

$\hat{U}_4^{\pm} = \hat{\tau}^{46} \pm i\hat{\tau}^{47}$

$\hat{Y}_4 = \frac{2}{\sqrt{3}}\hat{\tau}^{48} + \frac{1}{12}(I - 2\sqrt{6}\hat{\tau}^{415})$

$\hat{W}_4^{\pm} = \hat{\tau}^{49} \pm i\hat{\tau}^{410}$

$\hat{Z}_4^{\pm} = \hat{\tau}^{411} \pm i\hat{\tau}^{412}$

$\hat{z}_4^{\pm} = \hat{\tau}^{413} \pm i\hat{\tau}^{414}$

$\hat{C}_4 = 2(\hat{\tau}^{43})^2 - \frac{1}{\sqrt{3}}\hat{\tau}^{48} - \sqrt{\frac{8}{3}}\hat{\tau}^{415}.$

Njihovi komutatorji:

$\{\hat{\tau}^{43}, \hat{\tau}_4^{\pm}\}_- = \pm\hat{\tau}_4^{\pm}$ $\{\hat{\tau}_4^{+}, \hat{\tau}_4^{-}\}_- = 2\hat{\tau}^{43}$

$\{\hat{V}^{43}, \hat{V}_4^{\pm}\}_- = \pm\hat{V}_4^{\pm}$ $\{\hat{V}_4^{+}, \hat{V}_4^{-}\}_- = \hat{\tau}^{43} + \sqrt{3}\hat{\tau}^{48} := 2\hat{V}^{43}$

$\{\hat{U}^{43}, \hat{U}_4^{\pm}\}_- = \pm\hat{U}_4^{\pm}$ $\{\hat{U}_4^{+}, \hat{U}_4^{-}\}_- = -\hat{\tau}^{43} + \sqrt{3}\hat{\tau}^{48} := 2\hat{U}^{43}$

$\{\hat{W}^{43}, \hat{W}_4^{\pm}\}_- = \pm\hat{W}_4^{\pm}$ $\{\hat{W}_4^{+}, \hat{W}_4^{-}\}_- = \hat{\tau}^{43} + \frac{1}{\sqrt{3}}\hat{\tau}^{48} + 2\sqrt{\frac{2}{3}}\hat{\tau}^{415} := 2\hat{W}^{43}$

$\{\hat{Z}^{43}, \hat{Z}_4^{\pm}\}_- = \pm\hat{Z}_4^{\pm}$ $\{\hat{Z}_4^{+}, \hat{Z}_4^{-}\}_- = -\hat{\tau}^{43} + \frac{1}{\sqrt{3}}\hat{\tau}^{48} + 2\sqrt{\frac{2}{3}}\hat{\tau}^{415} := 2\hat{Z}^{43}$

$\{\hat{z}^{43}, \hat{z}_4^{\pm}\}_- = \pm\hat{z}_4^{\pm}$ $\{\hat{z}_4^{+}, \hat{z}_4^{-}\}_- = -\frac{2}{\sqrt{3}}\hat{\tau}^{48} + 2\sqrt{\frac{2}{3}}\hat{\tau}^{415} := 2\hat{z}^{43} := 2\hat{z}^{43}$

$\{\hat{\tau}^{43}, \hat{\tau}_4^{\pm}\}_- = \pm\hat{\tau}_4^{\pm}$ $\{\hat{Y}_4, \hat{\tau}_4^{\pm}\}_- = 0$ $\{\hat{C}_4, \hat{\tau}_4^{\pm}\}_- = 0$

$\{\hat{\tau}^{43}, \hat{V}_4^{\pm}\}_- = \pm\frac{1}{2}\hat{V}_4^{\pm}$ $\{\hat{Y}_4, \hat{V}_4^{\pm}\}_- = \pm\hat{V}_4^{\pm}$ $\{\hat{C}_4, \hat{V}_4^{\pm}\}_- = 0$

$\{\hat{\tau}^{43}, \hat{U}_4^{\pm}\}_- = \mp\frac{1}{2}\hat{U}_4^{\pm}$ $\{\hat{Y}_4, \hat{U}_4^{\pm}\}_- = \pm\hat{U}_4^{\pm}$ $\{\hat{C}_4, \hat{U}_4^{\pm}\}_- = 0$

$\{\hat{\tau}^{43}, \hat{W}_4^{\pm}\}_- = \pm\frac{1}{2}\hat{W}_4^{\pm}$ $\{\hat{Y}_4, \hat{W}_4^{\pm}\}_- = 0$ $\{\hat{C}_4, \hat{W}_4^{\pm}\}_- = \mp\hat{W}_4^{\pm}$

$\{\hat{\tau}^{43}, \hat{Z}_4^{\pm}\}_- = \mp\frac{1}{2}\hat{Z}_4^{\pm}$ $\{\hat{Y}_4, \hat{Z}_4^{\pm}\}_- = 0$ $\{\hat{C}_4, \hat{Z}_4^{\pm}\}_- = \mp\hat{Z}_4^{\pm}$

$\{\hat{\tau}^{43}, \hat{z}_4^{\pm}\}_- = 0$ $\{\hat{Y}_4, \hat{z}_4^{\pm}\}_- = \mp\hat{z}_4^{\pm}$ $\{\hat{C}_4, \hat{z}_4^{\pm}\}_- = \mp\hat{z}_4^{\pm}$

iz katerih je očitno, da zaprejo algebro.

Podgrupe grupe SU(4) so grupe SU(2), ki pa v našem primeru niso invariantne. Za njih pa lahko zapišemo:
$\{\hat{\tau}^i, \hat{\tau}^j\}_- = i\epsilon^{ijk}\hat{\tau}^k$, kjer $\hat{\tau}^i = \hat{\tau}^{4i}$ za $i, j, k = 1, 2, 3$
$\{\hat{V}^i, \hat{V}^j\}_- = i\epsilon^{ijk}\hat{V}^k$, kjer $\hat{V}^{i,j} = \hat{\tau}^{4i,4j}$ za $i, j = 4, 5$ oz. $\hat{V}^k = \hat{V}^{43}$
$\{\hat{U}^i, \hat{U}^j\}_- = i\epsilon^{ijk}\hat{U}^k$, kjer $\hat{U}^{i,j} = \hat{\tau}^{4i,4j}$ za $i, j = 6, 7$ oz. $\hat{U}^k = \hat{U}^{43}$
$\{\hat{W}^i, \hat{W}^j\}_- = i\epsilon^{ijk}\hat{\tau}^k$, kjer $\hat{W}^i = \hat{\tau}^{4i}$ za $i, j = 9, 10$ oz. $\hat{W}^k = \hat{W}^{43}$
$\{\hat{Z}^i, \hat{Z}^j\}_- = i\epsilon^{ijk}\hat{Z}^k$, kjer $\hat{Z}^{i,j} = \hat{\tau}^{4i,4j}$ za $i, j = 11, 12$ oz. $\hat{Z}^k = \hat{Z}^{43}$
$\{\hat{z}^i, \hat{z}^j\}_- = i\epsilon^{ijk}\hat{z}^k$, kjer $\hat{z}^{i,j} = \hat{\tau}^{4i,4j}$ za $i, j = 13, 14$ oz. $\hat{z}^k = \hat{z}^{43}$.

Vpeljemo novo bazo $|\tau^{43}Y_4C_4>$ za katero velja:
$\hat{\tau}^{43}|\tau^{43}Y_4C_4> = \tau^{43}|\tau^{43}Y_4C_4>$
$\hat{Y}_4|\tau^{43}Y_4C_4> = Y_4|\tau^{43}Y_4C_4>$
$\hat{C}_4|\tau^{43}Y_4C_4> = C_4|\tau^{43}Y_4C_4>$.

Kako pa ostali operatorji delujejo na bazi $|\tau^{43}Y_4C_4>$:
$\hat{\tau}^{43}\hat{\tau}_4^{\pm}|\tau^{43}Y_4C_4> = (\tau^{43} \pm 1)\hat{\tau}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{Y}_4\hat{\tau}_4^{\pm}|\tau^{43}Y_4C_4> = Y_4\hat{\tau}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{C}_4\hat{\tau}_4^{\pm}|\tau^{43}Y_4C_4> = C_4\hat{\tau}_4^{\pm}|\tau^{43}Y_4C_4>$

$\hat{\tau}^{43}\hat{V}_4^{\pm}|\tau^{43}Y_4C_4> = (\tau^{43} \pm \frac{1}{2})\hat{V}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{Y}_4\hat{V}_4^{\pm}|\tau^{43}Y_4C_4> = (Y_4 \pm 1)\hat{V}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{C}_4\hat{V}_4^{\pm}|\tau^{43}Y_4C_4> = C_4\hat{V}_4^{\pm}|\tau^{43}Y_4C_4>$

$\hat{\tau}^{43}\hat{U}_4^{\pm}|\tau^{43}Y_4C_4> = (\tau^{43} \mp \frac{1}{2})\hat{U}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{Y}_4\hat{U}_4^{\pm}|\tau^{43}Y_4C_4> = (Y_4 \pm 1)\hat{U}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{C}_4\hat{U}_4^{\pm}|\tau^{43}Y_4C_4> = C_4\hat{U}_4^{\pm}|\tau^{43}Y_4C_4>$

$\hat{\tau}^{43}\hat{W}_4^{\pm}|\tau^{43}Y_4C_4> = (\tau^{43} \pm \frac{1}{2})\hat{W}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{Y}_4\hat{W}_4^{\pm}|\tau^{43}Y_4C_4> = Y_4\hat{W}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{C}_4\hat{W}_4^{\pm}|\tau^{43}Y_4C_4> = (C_4 \mp 1)\hat{W}_4^{\pm}|\tau^{43}Y_4C_4>$

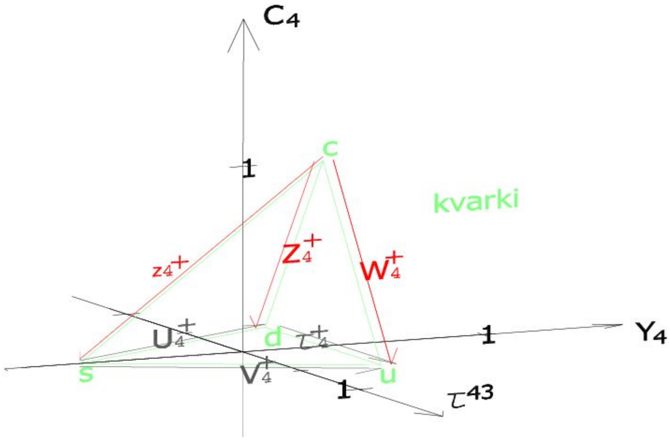
$\hat{\tau}^{43}\hat{Z}_4^{\pm}|\tau^{43}Y_4C_4> = (\tau^{43} \mp \frac{1}{2})\hat{Z}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{Y}_4\hat{Z}_4^{\pm}|\tau^{43}Y_4C_4> = Y_4\hat{Z}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{C}_4\hat{Z}_4^{\pm}|\tau^{43}Y_4C_4> = (C_4 \mp 1)\hat{Z}_4^{\pm}|\tau^{43}Y_4C_4>$

$\hat{\tau}^{43}\hat{z}_4^{\pm}|\tau^{43}Y_4C_4> = \tau^{43}\hat{z}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{Y}_4\hat{z}_4^{\pm}|\tau^{43}Y_4C_4> = (Y_4 \mp 1)\hat{z}_4^{\pm}|\tau^{43}Y_4C_4>$
$\hat{C}_4\hat{z}_4^{\pm}|\tau^{43}Y_4C_4> = (C_4 \mp 1)\hat{z}_4^{\pm}|\tau^{43}Y_4C_4>$

Oz.:

kvarki	τ_4^+	τ_4^-	V_4^+	V_4^-	U_4^+	U_4^-	W_4^+	W_4^-	Z_4^+	Z_4^-	z_4^+	z_4^-
$(\frac{1}{2}, \frac{1}{3}, 0) = \|u>$	0	d	0	s	0	0	0	c	0	0	0	0
$(-\frac{1}{2}, \frac{1}{3}, 0) = \|d>$	u	0	0	0	0	s	0	0	0	c	0	0
$(0, -\frac{2}{3}, 0) = \|s>$	0	0	u	0	d	0	0	0	0	0	0	s
$(0, \frac{1}{3}, 1) = \|c>$	0	0	0	0	0	0	u	0	d	0	s	0
antikvarki	τ_4^+	τ_4^-	V_4^+	V_4^-	U_4^+	U_4^-	W_4^+	W_4^-	Z_4^+	Z_4^-	z_4^+	z_4^-
$(-\frac{1}{2}, -\frac{1}{3}, 0) = \|\bar{u}>$	$-\bar{d}$	0	$-\bar{s}$	0	0	0	$-\bar{c}$	0	0	0	0	0
$(\frac{1}{2}, -\frac{1}{3}, 0) = \|\bar{d}>$	0	$-\bar{u}$	0	0	$-\bar{s}$	0	0	0	$-\bar{c}$	0	0	0
$(0, \frac{2}{3}, 0) = \|\bar{s}>$	0	0	0	$-\bar{u}$	0	$-\bar{d}$	0	0	0	$-\bar{d}$	$-\bar{c}$	0
$(0, -\frac{1}{3}, -1) = \|\bar{c}>$	0	0	0	0	0	0	0	$-\bar{u}$	0	0	0	$-\bar{s}$

Preslikave v prostoru (τ^{43}, Y_4, C_4) na primeru kvarkov:



Slika 6: Preslikave kvarkov z operatorji $\tau_4^\pm$, $V_4^\pm$, $U_4^\pm$, $W_4^\pm$, $Z_4^\pm$, $z_4^\pm$.

2.4 SU(4) multipleti

SU(4) multiplete zgradimo iz SU(3) multipletov oz. (T-Y)-multipletov (so vzoredni ravnini τ^{43}-Y); po analogiji gradnje multipletov SU(3):

$$[4 \otimes [\bar{4}] = ([3]^0 \oplus [1]^1) \otimes ([\bar{3}]^0 \oplus [1]^{-1}) = ([3] \otimes [\bar{3}])^0 \oplus [3]^0 \otimes [1]^{-1} \oplus [1]^1 \otimes [\bar{3}]^0 \oplus [1]^1 \otimes [1]^{-1} =$$

$$= [8]^0 \oplus [1]^0 \oplus [3]^{-1} \oplus [\bar{3}]^1 \oplus [1]^0 = [15] \oplus [1]$$

Kjer smo upoštevali, da je $[4] = [3]^0 \oplus [1]^1$, $[3] \otimes [\bar{3}] = [8] \oplus [1]$ ter da indeks v eksponentu predstavlja lastno vrednost operatorja $\hat{C}$.

Torej imamo SU(4) 15-plet in SU(4) singlet. 15-plet pa lahko seveda razbijemo na SU(3) multiplete: $[15] \rightarrow [1] \oplus [3] \oplus [\bar{3}] \oplus [8]$, kjer so:

$[1] \oplus [8]$=SU(3) nonet ($C_4 = 0$; singlet $\oplus$ oktet): $\eta^{`} \oplus \eta^0$, π^-, π^0, π^+, κ^0, $\bar{\kappa}^0$, κ^+, κ^-;

[3]= SU(3) triplet ($C_4 = 1$): D^0, F^+, D^+;

$[\bar{3}]$=SU(3) antitriplet ($C_4 = -1$): $\bar{D}^0$, F^-, D^-;

[1]= SU(4) singlet ($C_4 = 0$): η_c.

V središču prostora (τ^{43}, Y_4, C_4), v $(0,0,0)$, imamo štiri stanja , torej so štirikrat degenerirana. Dobimo jih na enak način kot smo dobili multiplete za grupo SU(3). Skratka:

$\underline{C_4 = 1}$		$\underline{C_4 = -1}$	
	$D^0 = \|c\bar{u}>$		$\bar{D}^0 = \|u\bar{c}>$
	$F^+ = \|c\bar{s}>$		$\bar{F}^- = \|s\bar{u}>$
	$D^+ = \|c\bar{d}>$		$\bar{D}^- = \|d\bar{c}>$

$\underline{C_4 = Y_4 = \tau^{43} = 0}$		$\underline{C_4 = 0; Y_4, \tau^{43} \text{ sicer}}$	
	$\pi^0 = \frac{1}{\sqrt{2}}(\|u\bar{u}> - \|d\bar{d}>)$		$\pi^- = \|d\bar{u}>$
	$\eta^0 = \frac{1}{2}(\|u\bar{u}> + \|d\bar{d}> - 2\|s\bar{s}>)$		$\pi^+ = \|u\bar{d}>$
	$\eta^{`} = \|u\bar{u}> + \|d\bar{d}> + \|s\bar{s}>$		$\kappa^0 = \|d\bar{s}>$
	$\eta_c = \|c\bar{c}>$		$\bar{\kappa}^0 = \|s\bar{d}>$
			$\kappa^+ = \|u\bar{s}>$
			$\kappa^- = \|s\bar{u}>$

3 DODATEK

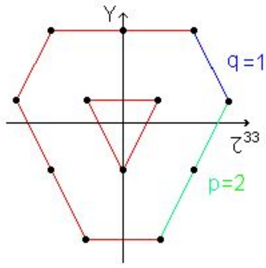
3.1 Stanja z maksimalno utežjo

<u>Def</u>. Stanje $|\tau^{33}Y>$ je stanje z maksimalno utežjo oz. $\psi_{max} = |\tau^{33}Y>_{max} > \ |\tau^{33'}Y'>$ natanko tedaj, ko $\tau^{33} > \tau^{33'}$ in za $\tau^{33} = \tau^{33'}$ velja $Y > Y'$.

Na sliki 3 na strani 6 je stanje z maksimalno utežjo za kvarke $(\frac{1}{2}, \frac{1}{3})$, za antikvarke pa stanje $(\frac{1}{2}, -\frac{1}{3})$.

Splošno lahko zapišemo stanje z maksimalno utežjo v obliki: $|\tau^{33}Y>_{max} = |\frac{p+q}{2}, \frac{p-q}{3}>$, kjer $(\hat{V}^-)^{p+1}\psi_{max} = 0$ in $(\hat{V}^-)^p\psi_{max} \neq 0$ ter $(\hat{T}^-)^{q+1}(\hat{V}^-)^p\psi_{max} = 0$ in $(\hat{T}^-)^q(\hat{V}^-)^p\psi_{max} \neq 0$.

p	q	
0	0	trivialno
1	0	triplet
0	1	antitriplet
1	1	mezoni



Slika 7: Določitev p, q.

3.2 Magnetni momenti hadronov

<u>Def</u>. Magnetni moment osnovnega stanja hadronov je vsota magnetnih momentov kvarkov: $\hat{\mu} = \sum_i \mu_q(i)\hat{\sigma}(i)$, kjer μ_q predstavlja magnetni moment posameznega kvarka, $\frac{1}{2}\hat{\sigma}(i)$ pa operator spina i-tega kvarka v hadronu.

Pričakovana vrednost z-komponente spina hadrona $|h>$ pa je: $\mu_h =< h|\hat{\mu}|h>$.

Za proton lahko zapišemo:

$$|p_\uparrow> = \frac{1}{\sqrt{18}}[2|u_\uparrow d_\downarrow u_\uparrow> + 2|u_\uparrow u_\uparrow d_\downarrow> + 2|d_\downarrow u_\uparrow u_\uparrow> - |u_\uparrow u_\downarrow d_\uparrow> - |u_\uparrow d_\uparrow u_\downarrow> - |u_\downarrow d_\uparrow u_\uparrow> -$$

$$-|d_\uparrow u_\downarrow u_\uparrow> - |d_\uparrow u_\uparrow u_\downarrow> - |u_\downarrow u_\uparrow d_\uparrow>]$$

za nevtron pa:

$$|n_\uparrow> = \frac{1}{\sqrt{18}}[2|d_\uparrow d_\uparrow u_\downarrow> + 2|u_\downarrow d_\uparrow d_\uparrow> + 2|d_\uparrow u_\downarrow d_\uparrow> - |d_\uparrow d_\downarrow u_\uparrow> - |d_\downarrow u_\uparrow d_\uparrow> - |u_\uparrow d_\uparrow d_\downarrow> -$$

$$-|d_\downarrow d_\uparrow u_\uparrow> - |d_\uparrow u_\uparrow d_\downarrow> - |u_\uparrow d_\downarrow d_\uparrow>]$$

$\mu_n =< n|\hat{\mu}|n> = \frac{1}{3}(4\mu_d - \mu_u)$,
$\mu_p =< n|\hat{\mu}|n> = \frac{1}{3}(4\mu_u - \mu_d)$.

Oz.
$\mu_d = \frac{1}{5}(\mu_p + 4\mu_n) = 1,852\mu_0$,
$\mu_u = \frac{1}{5}(4\mu_p + \mu_n) = -0,972\mu_0$,
kjer $\mu_0 = \frac{e\hbar}{2m_pc}$ (Števila so dobljene eksperimentalno).

3.3 Gell-Mann-Okubova masna formula za mezone

Če SU(3) simetrija ne bi bila kršena, potem bi imeli vsi mezoni v SU(3) multipletu enako maso. Seveda iz eksperimenta vemo, da temu ni tako. Torej pride do zloma SU(3) simetrije (je bolj zlomljena kot simetrija izospina).

Da ta zlom upoštevamo v Hamiltonjanu, moramo poleg starega invariantnega člena upoštevati še nek nov člen, ki povzroči zlom simetrije oz. premakne degenerirana stanja. Sestavimo ga lahko iz generatorjev grupe SU(3) (in ne iz Casimirjev; le-ti ne zlomijo simetrije).

Torej:
$\hat{H} = \hat{H}_i + \hat{H}_{ni}$,
$M =< \hat{H}_i > + < \hat{H}_{ni} >$,
$< \hat{H}_i >= m \;\; >> < \hat{H}_{ni} >$.

Na tem mestu zanemarimo masno razliko zaradi elektromagnetne interakcije oz. upoštevamo, da imajo vsi člani izospinskega multipleta enako maso oz. $\{\hat{H}_{ni}, \hat{\tau}^{33}\}_- = 0$.
Vemo, da z operatorjem $\hat{\tau}^{33}$ komutira $\hat{Y}$: $\hat{H}_{ni} = \alpha\hat{Y}$. Torej:

$$M = m + \alpha < \tau\tau^{33}Y|\hat{Y}|\tau\tau^{33}Y >= m + \alpha Y.$$

Ker smo zanemarili masno razliko zaradi elektromagnetne interakcije, pogledamo le delce z enakim nabojem oz. U-spin multiplete. Iz formule bi moralo veljati, da $M_{\eta^0} = M_{\pi^0}$, vendar eksperiment da: $M_{\eta^0} - M_{\pi^0} = 412,3MeV$. Zato moramo formulo popraviti.

Nov Hamiltonjan je sedaj: $\hat{H}^{`}_{ni} = \alpha\hat{Y} + \beta\hat{\tau}^2$,
nova masna formula pa $M = m + \alpha Y + \beta\tau(\tau + 1)$.

delec	Y	τ	M
κ	-1,1	$\frac{1}{2}$	$m + \alpha + \frac{3}{4}\beta$
η	0	0	m
π	0	1	$m + 2\beta$

Gell-Mann-Okubova masna formula, ki pa velja za vse SU(3) hadronske multiplete pa se glasi:

$$M = m + \alpha Y + \beta(\tau(\tau + 1) - \frac{1}{4}Y^2).$$

4 PRILOGA

4.1 Komponente simetričnega d^{ijk} ter antisimetričnega f^{ijk} tenzorja pri grupi SU(3)

Izračunajmo npr. f^{ijk} ter d^{ijk} za $i = 1$, $j = 5$, če vemo da:
$f^{ijk} = \frac{1}{4i} Tr(\{\lambda^i, \lambda^j\}_- \lambda^k)$ oz.
$d^{ijk} = \frac{1}{4} Tr(\{\lambda^i, \lambda^j\}_+ \lambda^k)$.

$$\{\lambda^1, \lambda^5\}_- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & -i & 0 \end{pmatrix} = -i\lambda^6$$

$$\{\lambda^1, \lambda^5\}_+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} = \lambda^7$$

Vemo, da: $Tr(\lambda^i \lambda^j) = 2\delta^{ij}$

Torej:
$f^{156} = \frac{1}{4i} Tr(\{\lambda^1, \lambda^5\}_- \lambda^6) = \frac{1}{4i} Tr(-i\lambda^6\lambda^6) = -\frac{1}{4} Tr(\lambda^6\lambda^6) = -\frac{1}{4}2 = -\frac{1}{2}$
$d^{157} = \frac{1}{4} Tr(\{\lambda^1, \lambda^5\}_+ \lambda^7) = \frac{1}{4} Tr(\lambda^7\lambda^7) = \frac{1}{4}2 = \frac{1}{2}$

4.2 Komutatorji generatorjev podgrup grupe SU(3) ter generatorji podgrup grupe SU(3) na bazi $|\tau^{33} Y >$

Za primer izračunajmo komutator $\{\hat{\tau}^+, \hat{\tau}^-\}_-$:

$\{\hat{\tau}^+, \hat{\tau}^-\}_- = \{\hat{\tau}^{31} + i\hat{\tau}^{32}, \hat{\tau}^{31} - i\hat{\tau}^{32}\}_- = i\{\hat{\tau}^{32}, \hat{\tau}^{31}\}_- - i\{\hat{\tau}^{31}, \hat{\tau}^{32}\}_- = 2i\{\hat{\tau}^{32}, \hat{\tau}^{31}\}_- = 2i(if^{213}\hat{\tau}^{33}) =$
$= 2i^2(-1)\hat{\tau}^{33} = 2\hat{\tau}^{33}$

Generatorji podgrup grupe SU(3) na bazi $|\tau^{33} Y >$:

$\hat{\tau}^{33}(\hat{\tau}^{\pm}|\tau^{33}\ Y >) = \hat{\tau}^{\pm}(\hat{\tau}^{33}|\tau^{33}\ Y >) \pm \hat{\tau}^{\pm}|\tau^{33}\ Y >= \hat{\tau}^{\pm}(\hat{\tau}^{33} \pm I)|\tau^{33}\ Y >= (\tau^{33} \pm 1)\hat{\tau}^{\pm}|\tau^{33}\ Y >$
$\hat{Y}(\hat{\tau}^{\pm}|\tau^{33}\ Y >) = \hat{\tau}^{\pm}(\hat{Y}|\tau^{33}\ Y >) = Y\hat{\tau}^{\pm}|\tau^{33}\ Y >$

Seveda je podobno za ostale operatorje.

4.3 Komutatorji med operatorji generatorjev podgrup grupe SU(3) za antidelce ter generatorji podgrup grupe SU(3) za antidelce na bazi $|\tau^{33} Y >$

Za primer izračunajmo komutator $\{\hat{\tau}_c^{33}, \hat{\tau}_c^{\pm}\}_-$:

$\{\hat{\tau}_c^{33}, \hat{\tau}_c^{\pm}\}_- = \{\hat{\tau}_c^{33}, \hat{\tau}_c^{31} \pm (-i)\hat{\tau}_c^{32}\}_- = i\hat{\tau}_c^{32} \pm i(-i)(-1)\hat{\tau}_c^{31} = i\hat{\tau}_c^{32} \mp \hat{\tau}_c^{31} = \mp(\hat{\tau}_c^{31} \mp i\hat{\tau}_c^{32}) = \mp\hat{\tau}_c^{\pm}$
$\{\hat{\tau}_c^{33}, \hat{\tau}_c^{\pm}\}_- = \{-\hat{\tau}^{33}, -\hat{\tau}^{31} \mp i\hat{\tau}^{32}\}_- = i\hat{\tau}^{32} \pm ii(-1)\hat{\tau}^{31} = i\hat{\tau}^{32} \pm \hat{\tau}^{31} = \pm(\hat{\tau}^{31} \pm i\hat{\tau}^{32}) = \pm\hat{\tau}^{\pm}$

Vidimo, da ponovno dobimo $\hat{\tau}_c^{\pm} = -\hat{\tau}^{\mp}$.

Generatorji podgrup grupe SU(3) na bazi $|\tau^{33} Y >$:

$\hat{\tau}_c^{33}(\hat{\tau}_c^{\pm}|\tau^{33}\ Y >) = \hat{\tau}_c^{\pm}(\hat{\tau}_c^{33}|\tau^{33} Y >) \mp \hat{\tau}_c^{\pm}|\tau^{33}\ Y >= (\tau_c^{33} \mp 1)\hat{\tau}_c^{\pm}|\tau^{33}\ Y >= (-\tau^{33} \mp 1)(-\hat{\tau}^{\pm})|\tau^{33}\ Y >=$
$= (\tau^{33} \pm 1)\hat{\tau}^{\mp}|\tau^{33}\ Y >$

Kaj pa generatorji za antidelce naredijo na stanjih npr. $(-\frac{1}{2}, -\frac{1}{3})$:

$\hat{\tau}_c^{+}|\bar{u} >= \hat{\tau}_c^{+}(-\frac{1}{2}, -\frac{1}{3}) = (\frac{1}{2}, -\frac{1}{3}) = |\bar{d} >$
$-\hat{\tau}^{+}|\bar{u} >= |\bar{d} >$
$\hat{\tau}^{+}|\bar{u} >= -|\bar{d} >$

Seveda je podobno za vse ostale operatorje.

Kazalo

Literatura

[1] Greiner Müller *Quantum Mechanics Symmetries*